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ABSTRACT

A set of three teacher-prepared Learning Activity Packages (LAPs) in geometry, the units cover the topics of distance, lines, planes, separation; angles and triangles; and congruences. The units each include a rationale for the material, a list of behavioral objectives, a list of resources including texts (with reading assignments and problem sets specified) and tape recordings, a student self-evaluation sheet, suggestions for advanced study, and references. For other documents in this series, see SE 015 193, SE 015 194, SE 015 195, and SE 015 196. (DT)

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LEARNING ACTIVITY PACKAGE

DISTANCE, LINES, PLANES, SEPARATION

Ninety Six High School
Ninety Six, S.C.

Geometry 114

VIEWED BY

LAP NUMBER 26

WRITTEN BY Bill Holland

RATIONALE

DISTANCE, LINES, PLANES, SEPARATION

The SYSTEM of GEOMETRY is an historically important one and is essentially the system as first created by Euclid. However, the particular development followed here is not truly Euclidian. You will use your knowledge of the set of real numbers since Geometry is introduced as a mathematical study of sets of points, and geometric figures are treated as such.

BASIC POSTULATES about distance are expressed in terms of one-to-one correspondence between sets of real numbers and sets of points. The equality and order properties of a field are used to develop concepts of distance, betweenness, and congruence. The absolute value of a real number is necessary for a concise and mathematically accurate statement of how to compute the distance between two points on a number line.

The structure of a postulated system is discussed and the necessity for using precise language is stressed! As you work with definitions, postulates, and theorems, they will become more meaningful to you. It is important to clarify the use of the three undefined terms - POINTS, LINE and PLANE!!

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

1. Given any set:

   a. Write a word description of the set.
   b. Write a description using the roster method.

2. Given any pair of sets:

   a. Determine if they are equal.
   b. Determine if one set is a subset of the other.
   c. Name their intersection.
   d. Name their union.
   e. Determine if they intersect.

3. Given a set of real numbers, state which elements are:

   a. integers
   b. rational numbers
   c. irrational numbers

4. Given statements, using the relations listed below, determine whether they are TRUE or FALSE:

   a. $<$ (less than)
   b. $>$ (greater than)
   c. $\leq$ (less than or equal to)
   d. $\geq$ (greater than or equal to)

5. Given a list of statements, determine which ones are examples of the following properties:

   a. The Trichotomy Property
   b. The Transitive Property of Order
   c. The Property Related to the Taking of Opposites
   d. Addition for Inequalities
   e. The Property Connecting Order with Equality
   f. Multiplication for Inequalities

6. Decide for what replacement of x, the real number -x will name:

   a. the number zero
   b. a positive real number
   c. a negative number

7. Given a real number, decide whether or not it has a positive square root.

8. Given a non-negative real number, name its positive square root and its negative square root.

9. Given any real numbers x and y, determine the following:

   a. $|x|$
   b. $|x + y|$
   c. $|x| + |y|$
   d. $|x - y$
   e. $|x| - |y|$

10. Be able to graph algebraic sentences of the following forms:

   a. $|x| < a$

   b. $|x| > a$

   c. $|x| = a$

   d. $|x| \leq a$

   e. $|x| \geq a$

SECTION I

RESOURCES

I. Readings:

1. Moise: Geometry - #1, #2 pp. 1-3, 8-11, 15-17; #3 - #8 pp. 21-24; #9 - #10 pp. 26.

2. Jurgensen: Modern Geometry - #1, #2 pp. 1-3, 5-7, 8-10; #3 - #8 pp. 11-12, 14-17; #9 - #10 pp. 18-20.

3. Nichols: Modern Geometry - #1, #2 pp. 1-8; #3 - #8 pp. 16-21 (objectives 6, 7, and 8 not covered); #9, #10 p. 22 (objective 10 not covered).

4. Anderson: School Mathematics Geometry - #1, #2 pp. 17-19, 21-23; #3 - #8 pp. 25-28, 29-30; #9, #10 pp. 29-31 (objective 10 not covered).

5. Lewis: Geometry - #1, #2 pp. 10-11; #3 - #10 ____.

II. Problems:

1. Moise: Geometry - #1, #2 pp. 4-8 ex. 1-3, 6-13, pp. 12-13 ex. 1, 3-5, pp. 18-20 ex. 1-12, 14-15, p. 50 ex. 1-2; #3 - #8 pp. 24-25 ex. 1-9, pp. 50-51 ex. 3, 5, 12; #9, #10 p. 27 ex. 1-6, p. 50 ex. 4.

2. Jurgensen: Modern Geometry - #1, #2 pp. 3-5 ex. 1-36, pp. 7-8 ex. 1-32 pp. 10-11 ex. 1-16 p. 45 ex. 1-10, p. 47 ex. 1-15; #3 - #8 pp. 12-13 ex. 1-36, p. 17 ex. 1-36, pp. 47-48 ex. 16-20; #9, #10 p. 46 ex. 19-22, p. 47 ex. 16-20, 27-29, 35, 37, 39, 45, pp. 20-21 ex. 1-40.

3. Nichols: Modern Geometry - #1, #2 p. 8 ex. 1-6; #3 - #8 pp. 18-19 ex. 1 (a-l), 10, pp. 20-21 ex. 1(a-l), 6, 8 (a-h), 10; #9, #10 p. 22 ex. 1-3.

4. Anderson: School Mathematics Geometry - #1, #2 pp. 20-21 ex. 1-13, pp. 23-24 ex. 1-9; #3 - #8 pp. 28-29 ex. 1-11, pp. 32-33 ex. 1-14, 26-28; #9, #10 pp. 32-33 ex. 15-25, 29-31.

5. Lewis: Geometry - #1, #2 pp. 11-13 ex. 1-12; #3 - #10 ____.

SELF-EVALUATION I

1. Match the statement in column A with its equivalent statement in column B.

COLUMN A	COLUMN B
1. set of numbers which are exponents in the expression: $5x^7 + 4x^5 + 7x^3$	a. $\emptyset$
2. set of numbers which are coefficients in the above expression	b. the days of the week whose first letters are not "T"
3. {Sun., Mon., Wed., Fri., Sat.,}	c. {3, 5, 7}
4. the set of odd numbers whose squares are even numbers	d. the set of even integers less than nine and greater than one
5. {2, 4, 6, 8}	e. {4, 5, 7}

2. Answer the following true or false:

_____ a. The subsets of {a} are {a} and $\emptyset$.

_____ b. Sets A and B are equal if A is the set of all whole numbers between $\frac{3}{2}$ and $\frac{22}{3}$ and B is the set whose elements are 2, 3, 4, 5, 6, 7.

_____ c. The union of the set of primes and the set of positive even integers is the set of all positive integers.

_____ d. It is possible for the intersection of a line and a plane to be the empty set.

_____ e. If two lines intersect, then their intersection is never the empty set.

_____ f. The intersection of the set of real numbers and the set of integers is the set of integers.

3. Given A = $\{-3, 0, 0.25, \pi, \sqrt{2}, 3, 0.\overline{82}, 0.763..., -1\}$

a) Which of the elements of A are rational numbers?

b) Which of the elements of A are irrational numbers?

c) Which element in A is greater than each of the other elements?

d) Which element in A is smaller than each of the others?

e) Between which two integers does $\sqrt{2}$ lie?

f) Between which two integers does π lie?

4. Write the following, using the symbols of order, $<$, $\leq$, $>$, $\geq$.

a) x is a number greater than 0.

b) y is a number between -1 and 2.

c) w is a number between -2 and 1 inclusive.

d) p is a positive number.

e) n is a negative number.

f) n is a non-negative number.

5. Matching. Select a letter of the name of the property in column B which matches the number of the example of the property in column A. (You may use the property more than once.)

COLUMN A	COLUMN B
a) If $r \leq s$ and $s < t$, then $r < t$	1) Trichotomy Property
b) If $r < s$ and $a > 0$, then $ar < as$	2) Transitive Property of Order
c) If $r < s$, then $r + y < s + y$.	3) Property Related to the Taking of Opposites
d) If $r < s$, then $-3r > -3s$.	4) Addition for Inequalities
e) If $r \neq 3$ and $r \neq 3$, then $r > 3$	5) Property Connecting Order with Equality
f) If $r + 3 = s$, then $r < s$.	6) Multiplication for Inequalities
g) If $r < 3$, then $-r > -3$.	

6. Write the simplest real number name for each of the following:

a) $|x|$

b) $\sqrt{\frac{169}{529}}$

c) a square root of 81

d) $\sqrt{-4}$

e) $|0|$

f) $|-\sqrt{19}|$

g) $|13| - |17|$

h) $|13 - 17|$

i) $|-13| + |17|$

7. Graph the following algebraic statements (geometric interpretation):

   a) $|x| = 3$

   b) $|x| > 3$

   c) $|x| < 3$

   d) $|x| \geq 3$

IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCED STUDY I

1. Determine the solution sets of the following:

   a) $|x - 3| \leq 6$

   b) $|5x - 9| > 2$

   c) $x^2 - 6x + 9 < 5$

   d) $x^2 - 11x + 10 \geq 1$

   e) $\frac{x + 1}{x - 3} \leq 5$

   f) $\frac{x + 7}{2x - 6} > 3$

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

11. Decide whether or not given statements are true or false when the statements involve the following Theorems, Definitions, and Postulates:

    a. Distance Postulate
    b. Ruler Postulate
    c. Definition of a Coordinate System
    d. Ruler Placement Postulate
    e. Definition of Between
    f. Line Postulate
    g. Definition of a Segment
    h. Definition of Length of a Segment
    i. Definition of a Ray
    j. Definition of Opposite Rays
    k. The Point-Plotting Theorem
    l. Definition of Midpoint of a Segment
    m. Definition of a Bisector
    n. Theorem: Every segment has exactly one midpoint
    o. Definition of Absolute Value
    p. Definition of "$\sqrt{\ }$"

12. Given the coordinates of two points, compute the distance between the two points.

13. Given the coordinates of two points, compute the coordinate of the midpoint of the segment.

14. Given a coordinate system on a line with the coordinates of two points stated, apply the Ruler Placement Postulate and assign 0 to one of the points and determine the positive coordinate of the other point.

15. Decide whether or not given statements are true or false when they involve the following Theorems, Postulates, and Definitions:

    a. Definition of Space
    b. Definition of COllinear and Coplanar
    c. Theorem: If two different lines intersect, their intersection contains only one poitn.
    d. Postulate: If two points of a line lie in a plane, then the line lies in the same plane.
    e. Theorem: If a line intersects a plane not containing it, then the intersection contains only one point.
    f. The Plane Postulate
    g. Theorem: Given a line and a point not on the line, there is exactly one plane containing them.
    h. Theorem: Given two intersecting lines, there is exactly one plane containing both.

i. Postulate: If two different planes intersect, then their intersection is a line.
j. Definition of a Convex Set
k. Definition of Separation
l. Plane and Space Separation Postulates
m. Definitions of Half-line, Half-plane, and Half-space

16. Given a drawing showing Points, Lines, Planes, and a list of Sets, determine which of the sets contain members which are:

    a. Collinear
    b. Coplanar
    c. In the same Half-space
    d. In the opposite Half-plane

17. Given any pair of geometric figures (Points, Lines, Planes), name their intersection.

18. Determine from a given set of drawings which ones represent convex sets.

RESOURCES II

I. Readings:

1. Moise: Geometry - #11-#14 pp. 31, 33-35, 38-39, 41-43, 44-45, 46-48; #15-#18 pp. 55-58, 59-61, 63-65.

2. Jurgensen: Modern Geometry - #11-#14 pp. 14-16, 18-19, 22, 26-28; #15-#18 pp. 22-23, 26, 31.

3. Nichols: Modern Geometry - #11-#14 pp. 22-25, 26-27, 28-29, 32-34; #15-#18 pp. 30-31, 40.

4. Anderson: School Mathematics Geometry - #11-#14 pp. 43-46, 49-52, 54-57; #15-#18 pp. 69-70, 74-76, 84-86, 88.

5. Lewis: Geometry - #11-#14 pp. 13-17, 22-26, 37-39; #15-#18 ____.

II. Problems:

1. Moise: Geometry - #11-#14 pp. 31-32 ex. 1-5, pp. 35-37 ex. 1-11, pp. 39-40 ex. 1-7, pp. 43-44 ex. 1-4, 6-14, pp. 45-46 ex. 1-7, pp. 48-49 ex. 1-5; #15-#18 pp. 58-59 ex. 1-7, pp. 61-62 ex. 1-12, pp. 66-67 ex. 1-15.

2. Jurgensen: Modern Geometry - #11-#14 pp. 20-21 ex. 1-18, pp. 23-24 ex. 1-12, pp. 29-30 ex. 1-12, 14-15, 19-20, 23-24; #15-#18 pp. 25-26 ex. 13, 16-18, 21-22, 25-32.

3. Nichols: Modern Geometry - #11-#14 pp. 25-26 ex. 1-11, p. 27 ex. 1-8, pp. 29-30 ex. 1-9, pp. 34-35 ex. 1-7; #15-#18 pp. 31-32 ex. 1-5, p. 41 ex. 8-12.

4. Anderson: School Mathematics Geometry - #11-#14 pp. 47-49 ex. 1-10, pp. 53-54 ex. 1-14, pp. 58-59 ex. 1-5; #15-#18 pp. 72-74 ex. 1-13, p. 77 ex. 1-10, pp. 87-88 ex. 1-11, pp. 90-91 ex. 1-10.

5. Lewis: Geometry - #11-#14 pp. 18-20 ex. 1-10, pp. 26-27 ex. 1-6, pp. 39-40 ex. 1, 3, 5, 7, 9.

III. Audio:

Objectives 15-18: Wollensak Teaching Tape R-3605

SELF-EVALUATION II

1. Answer the following True or False:

_____ a. The distance between any two points can never be a negative real number.

_____ b. Every point on a number line corresponds to a rational number.

_____ c. The number assigned to a point is called the coordinate of the point.

_____ d. If PQ = 0, then P and Q are two names for the same point.

_____ e. The distance between two points is the absolute value of the difference of their coordinates.

_____ f. If AB + BC = AC and A, B, and C are different points on the same line, then B is between A and C.

_____ g. Given a coordinate system on a line, if the coordinate of point R is -7 and the coordinate of point S is 3, then RS = 4.

_____ h. Given a coordinate system on a line, if the coordinate of point A is 8, the coordinate of point B is 20, and C is the midpoint of $\overline{AB}$, then the coordinate of C is 12.

_____ i. If $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays, then C is between A and B.

_____ j. If A and B are two different points, then AB must name a positive real number.

2. Complete each statement with the correct symbol or word:

a) A, B, and C are three points on a line. If AC = 7 and BC = 5, and B is between A and C, then AB = _____?

b) If 2 is the coordinate of A and -7 is the coordinate of B, then AB = _____?

3. a) C, the midpoint of $\overline{AB}$, has coordinate 5. If the coordinate of A is greater than the coordinate of B, and BC = 9, what are the coordinates of A and B?

b) If A is -3 and C is 11, then the midpoint B corresponds to _____.

4. Given a coordinate system on line RS. If the Ruler Postulate is applied with R corresponding to 0, what is the new coordinate of S when:

   a) R is -7 and S is 6.

   b) R is 3 and S is 11.

   c) R is -5 and S is y.

5. Decide whether or not the following statements are true or false.

   _____ a. A line and a plane always have a point in common.

   _____ b. Two lines always lie on the same plane.

   _____ c. There are lines which do not intersect each other.

   _____ d. If three points are collinear, they are coplanar.

   _____ e. A point and a line always lie in one and only one plane.

   _____ f. Given two different points A and B, there are at least two different lines that contain both A and B.

   _____ g. A line has two end points.

   _____ h. Given two points, there is more than one plane containing them.

   _____ i. Given two points A and B, if A $\in$ line L_1 and B $\in$ line L_2 and $\overline{AB} \leq L$, then $L_1 = L_2$.

   _____ j. There exists three lines in a plane such that they separate the plane into seven regions.

   _____ k. Given: Points A, B, and C lie in plane P. Points A, B, and C lie in plane R. You can conclude that plane F is the same plane as plane R.

   _____ l. A circle is a convex set.

   _____ m. Two half-planes with a common edge are coplanar.

   _____ n. A half-space is a convex set of points.

   _____ o. We have a postulate stating that two intersecting lines determine a plane.

   _____ p. There are three conditions that determine a plane.

   _____ q. Given a line L, if L $\subseteq$ plane R and L $\subseteq$ plane S, then R = s.

6. Study the three dimensional figure given (in which R, S, T, U are coplanar) and answer the following questions:

   a) Are R, S, T, U, and X coplanar?

   b) Are V, U, and W collinear?

   c) Are V, T, X, and R coplanar?

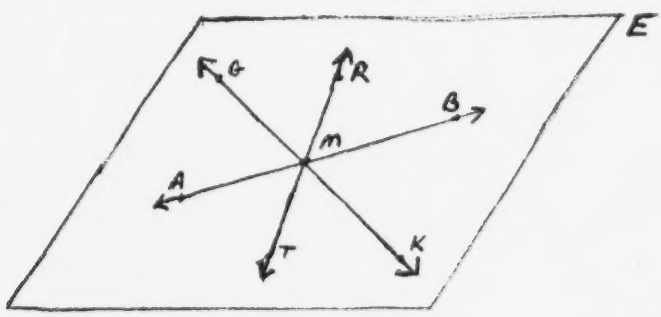
   d) Are V, T, S, and W coplanar?

   e) Are V, S, and W coplanar?

   f) Are V, U, R, and W coplanar?

   g) Does the interior of the figure represent a convex set of points?

7. Write a word or phrase which would make each of the following a complete and true statement. In the figure below, plane E contains lines $\overleftrightarrow{AB}$, $\overleftrightarrow{GK}$, and $\overleftrightarrow{RT}$, all of which intersect at M.



   a) According to the _____ Postulate, since T and R are in different half-planes formed by $\overleftrightarrow{AB}$, then $\overline{TR}$ must _____.

   b) Points A and B are said to lie on _____ sides of $\overleftrightarrow{GK}$.

   c) Points A and G lie on the same side of $\overleftrightarrow{TR}$ because because _____ _____.

   d) According to one of our Postulates, if we know only that the given plane contains A and _____.

ADVANCED STUDY II

1. An airplane is 4 miles east and 2 miles north of Baltimore. Five minutes later it is 15 miles east and 2 miles north of Baltimore. How far and how fast did it travel in the 5 minutes?

2. Airplane B is 4 miles east and 2 miles north of Baltimore. In four minutes the plane traveled 12 miles due east. How far east and how far north is the plane from Baltimore now? What was its rate of speed?

3. A boat is traveling from buoy #4 to buoy #6. Buoy #4 is located 6 miles east and 2 miles south of a lighthouse. Buoy #6 is located 16 miles east and 4 miles south of the lighthouse. When the boat completes half of its trip, how far south of the lighthouse will it be?

REFERENCES

I. Textbooks:

1. Moise, Downs: Geometry (Addison-Wesley Publishing Co., Inc., 1964).

2. Jurgensen, Donnelly, Dolciani: Modern Geometry (Houghton-Mifflin Co., 1965).

3. Nichols, Palmer, Schacht: Modern Geometry (Holt, Rinehart, and Winston, Inc., 1968).

4. Anderson, Garon, Gremillion: School Mathematics Geometry (Houghton-Mifflin Co., 1968).

5. Lewis: Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968).

II. Audio:

1. Wollensak Teaching Tapes: R-3605

LEARNING ACTIVITY PACKAGE

ANGLES AND TRIANGLES

Ninety Six High School - Ninety Six, S.C.

Geometry 114

LAP NUMBER 27

WRITTEN BY Bill Holland

EWED BY

92172

RATIONALE

ANGLES AND TRIANGLES

We are sure that most of you can draw pictures of angles and triangles? In science we refer to angles in relation to refraction and reflection of light, to the bonding of angles in molecular structure, and to triangulation in determing distance to a star. But, if we were to ask you to give a precise definition of angles and triangles, you might have some difficulty. In this section we will discuss the basic definitions of these geometric figures. The idesa we thus communicate to you will be used as a basis for proving some theorems later on in this LAP.

Many of the properties of triangles and angles are dependent on the measures of these geometric figures. We will discuss measurement of angles, including how to measure angles and what postulates allow us to measure them. We will also refer to some of the special relationships, based upon measure, which exist between angles.

You will find that we can classify any angle into one of three categories. There will be no new postulates in this section, but we will be using our previous definitions and postulates to prove a few theorems about angles. It is not necessary for you to memorize the proofs of these theorems, but it is recommended that you study them and try to understand the thinking behind the proofs, and become aware of how to develop your own simple proofs.

SECTION I

Behavioral Objectives:

By the completion of the prescribed course of study, you will be able to:

1. Given a drawing of an angle, properly labeled,
   a. Name the vertex of the angle.
   b. Name the sides of the angle.
   c. Name the angle.
2. Given a drawing of a triangle, properly labeled,
   a. Name the sides of the triangle.
   b. Name the angles of the triangle.
   c. Name the triangle.
   d. Identify an included side.
   e. Identify an included angle.
   f. Identify a side opposite a given angle.
   g. Identify an angle opposite a given side.
3. Given a drawing of an angle, properly labeled, and any point, decide whether the point lies in the interior of the angle, in the exterior of the angle, or on the angle itself.
4. Given a drawing of a triangle, properly labeled, and a word description of a point, decide if the point lies in the interior of the triangle, the exterior of the triangle, or on the triangle itself.
5. Answer correctly true-false, matching, or completion type questions involving:
   a. The definition of an angle.
   b. The definition of a triangle.
   c. The definition of the interior of an angle.
   d. The definition of the exterior of an angle.

e. The definition of the interior of a triangle.

f. The definition of the exterior of a triangle.

6. Given a drawing of one or more angles, estimate the measure of the angles to the nearest 10 degrees.

7. Given a composite drawing of angles, classify each of the angles as acute, right, or obtuse.

8. Given a drawing of a pair of lines, rays, or segments; determine whether or not the pair of geometric figures are perpendicular.

9. Given a measure of an angle, compute the measure of its supplement.

10. Given a relationship between two supplementary angles, compute the measure of the given angles through the use of the definition of supplementary angles and the supplement postulate.

11. Given the measure of angle, compute the measure of its complement.

12. Given a relationship between two complementary angles, compute the measure of the given angles through the use of the definition of complementary angles.

13. Answer correctly true-false, matching, or completion type questions involving:

a. The angle Measurement Postulate.

b. The definition of the measure of an angle.

c. The Angle Construction Postulate.

d. The Angle Addition Postulate.

e. The definition of a linear pair.

f. The definition of supplementary angles.

g. The Supplement Postulate.

14. Answer correctly true-false matching or completion type questions involving:

a. Acute angles

b. Obtuse angles

c. Right angles

d. The definition of a ray, a segment, or perpendicular lines.

e. The definition of complementary angles.

f. The definition of vertical angles.

g. The definition of adjacent angles.

h. Postulates and theorems relating to angles and their relationship to other angles.

15. Given a drawing of intersecting lines and rays, and a list of angle relationships; match a pair of angles with the correct relationship involving:

a. Acute angles

b. Obtuse angles

c. Right angles

d. Equal angles

e. Supplementary angles

f. Complementary angles

g. Linear pair

h. Vertical angles

i. Congruent angles

j. Adjacent angles

SECTION I

Resources

I. Readings:

1. Moise: Geometry - #1 - #5 pp. 75-77; #6 - #8 pp. 80-82, 87-88; #9 - #12 pp. 83, 87; #13 - #15 pp. 82-83, 88-92.

2. Jurgensen: Modern Geometry - #1 - 5 pp. 30-31; #6 - #8 pp. 33-36, 39, 125; #9 - #12 pp. 40, 130-131, 133; #13 - #15 pp. 40, 125, 131, 138-139.

3. Nichols: Modern Geometry - #1 - #5 pp. 38-40, 42-43, 47-48; #6 - #8 pp. 111-115, 121-122; #9 - #12 pp. 49-50, 59-60; #13 - #15 pp. 118-121, 125-127.

4. Anderson: School Mathematics Geometry - #1 - #5 pp. 99-104, 107-108; #5 - #8 pp. 111-115, 121-122; #9 - #12 pp. 115, 121-122; #13 - #15 pp. 18-121, 125-127.

5. Lewis: Geometry - #1 - #5 pp. 17-18; #6 - #8 pp. 27-31, 33-34; #9 - #12 pp. 31-32; #13 - #15 pp. 59-61, 63.

II. Problems:

1. Moise: Geometry - #1 - #5 pp. 78-79 ex. 1 - 18; #6 - #8 pp. 84-85 ex. 6-10, pp. 88-89 ex 1, 3, 5; #9 - #12 p. 84 ex. 1-5, pp. 85-86 ex. 13, 15-18, pp. 88-89 ex. 1-2, 6-8; #13 - #15 pp. 85-87 ex. 12, 14, 20, p. 94 ex. 1-8.

2. Jurgensen: Modern Geometry - #1 - #5 p. 32 ex. 1-12; #6 - #8 p. 38 ex. 1-22, p. 129 ex. 1-20, p. 141 ex. 1-10; #9 - #12 pp. 42-43 ex. 1-34, p. 135 ex. 1-10, p. 141 ex. 11-19, p. 143 ex. 39-42; #13 - #15 p. 129 ex. 1-20, pp. 135-136 ex. 11-14, 19-22, p. 141 ex. 1-14.

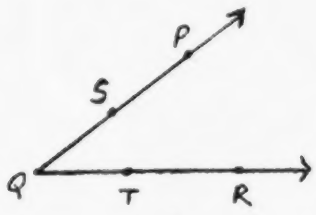
3. Nichols: Modern Geometry - #1 - #5 p. 41 ex. 1-6, pp. 43-44 ex. 1-10, pp. 48-49 ex. 1-4; #6 - #8 p. 46 ex. 1-3, p. 58 ex. 1-6; #9 - #12 pp. 50-51 ex. 1-8, p. 60 ex. 1-9, pp. 78-79 ex. 4-5, 8-12; #13 - #15 pp. 53-54 ex. 1-10, p. 79 ex. 7.

4. Anderson: School Mathematics Geometry - #1 - #5 pp. 106-107 ex. 1-8, p. 110 ex. 1-12; #6 - #8 pp. 116-117 ex. 1-8, pp. 123-124 ex. 1-3, 10-11; #9 - #12 p. 118 ex. 9-15, p. 123 ex. 3, 5-7; #13 - #15 p. 138 ex. 1-2, 6, 11.

5. Lewis: Geometry - #1 - #5 p. 19 ex. 3-5; #6 - #8 pp. 32-33 ex. 1-3, 8-10, pp. 34-35 ex. 1-8; #9 - #12 p. 33 ex. 4-7, 11; #13 - #15 p. 61 ex. 1-2, p. 64 ex. 4-8, pp. 67-68 ex. 3-4, 6, 8, 10.

SELF-EVALUATION I

1. Which of the following is <u>not</u> a name for ∠Q in the illustration?



____ a) ∠PQR

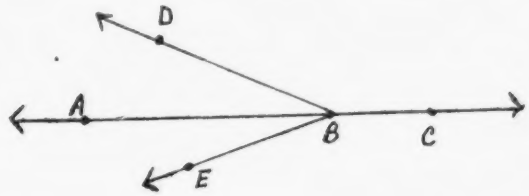
____ b) ∠TQS

____ c) ∠QSR

____ d) ∠TQP

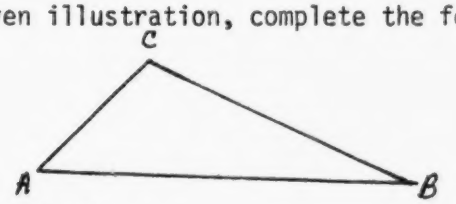
2. In the figure at the right, A, B, and C are collinear:

a) Name five angles



b) What is the vertex of all the angles named?

3. Using the given illustration, complete the following:



a. Name the triangle.

b. Name the angles of the triangle.

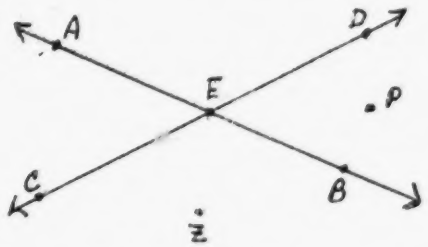
c. The side opposite ∠A is _____.

d. The angle included by sides $\overline{AB}$ and $\overline{BC}$ is _____.

e. The side included by ∠A and ∠C is _____.

f. The angle opposite $\overline{AC}$ is _____.

4. For the following diagram, classify each of the following statements as True or False.



_____ a. Z is in the interior of ∠BED.

_____ b. P is in the interior of ∠BED.

_____ c. P belongs to ∠BED.

_____ d. A belongs to ∠AED.

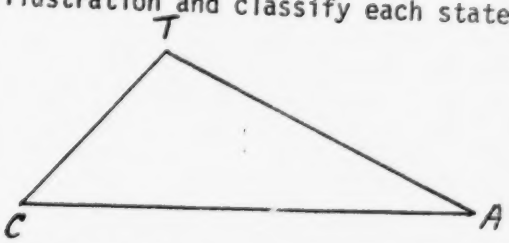
_____ e. Z is in the exterior of ∠AED.

_____ f. B belongs to ∠AEC.

_____ g. E belongs to all four angles illustrated.

_____ h. D is in the exterior of ∠AEC.

5. Use the following illustration and classify each statement true or false:



_____ a. If P is in the same half-plane with edge $\overleftrightarrow{AC}$ as T, and in the same half-plane with edge $\overleftrightarrow{CT}$ as A, then P is in interior of ∠C.

_____ b. If P is located as described in (a), then P is in the interior of ΔTCA.

_____ c. If P is in the interior of both ∠C and ∠A, then P is in the interior of △TCA.

6. Complete the following statements by selecting the one best word or phrase to make each a true statement:

A. An angle is the _____ of two _____ which have the end-point and do not lie in the same _____.

B. If A, B, and C are any _____ points, then the _____ of segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ is a triangle.

C. The _____ of ∠BAC is the set of all points in plane E that do not lie in the interior of ∠BAC or on the _____.

D. The interior of an angle is the _____ of two _____.

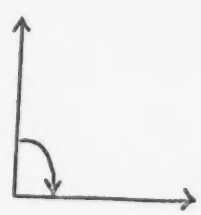
E. A point lies in the interior of a triangle if it lies in the _____ of each _____ of the triangle.

F. Which of the following is not a convex set? _____

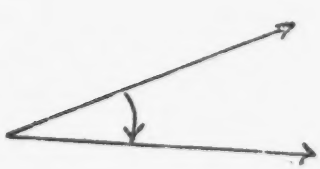
(1) △RST (2) half-plane (3) $\overleftrightarrow{MN}$ (4) interior of △MAD

7. Matching: Estimate the size of the angles without using a protractor. Decide which of the angles shown have measures within the indicated ranges listed at the right.

___ (a)



___ (b)



(1) $15 < x < 35$

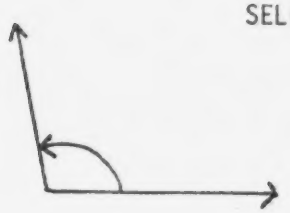
(2) $70 < x < 90$

(3) $80 < x < 100$

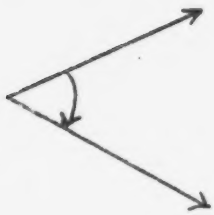
(4) $45 < x < 60$

(5) $90 < x < 110$

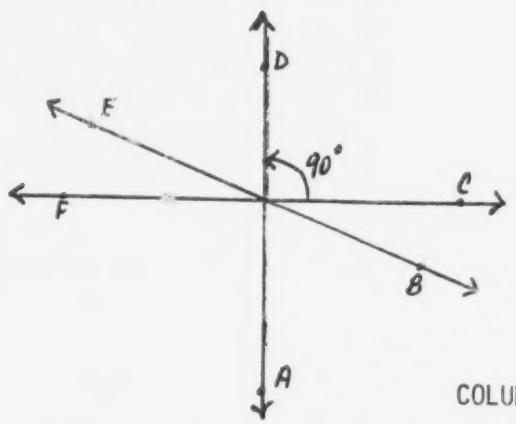
___ (c)



___ (d)

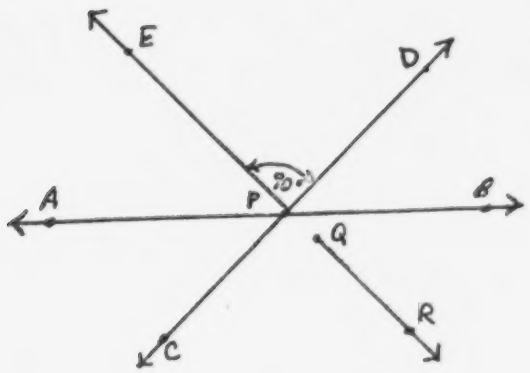


8. Matching: Use the illustration below and correctly match the items of column B with items in column A.



COLUMN A	COLUMN B
___ a) ∠DGF	1) acute angle
___ b) ∠AGE and ∠DGB	2) obtuse angle
___ c) ∠BGA	3) right angle
___ d) ∠EGC	4) vertical angles
___ e) ∠EGF and ∠FGA	5) none of the above

9.



Using the diagram decide which of the following statements is true or false.

_____ (a) If $\overrightarrow{PE} \perp \overleftrightarrow{CD}$, then m ∠CPA + m ∠DPE.

_____ (b) $\overrightarrow{QR}$ is perpendicular to $\overleftrightarrow{CD}$.

_____ (c) $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{ER}$.

_____ (d) $\overrightarrow{PE}$ is perpendicular to $\overrightarrow{PC}$.

_____ (e) $\overline{EP}$ is perpendicular to $\overline{AB}$.

10. Determine the measure of the supplement of an angle whose measure is:

(a) 75

(b) 68

(c) 35.5

(d) m + y

(e) 180 - x

(f) 90 - y

11. a) Twice the measure of an angle is 30 less than five times the measure of its supplement. What is the measure of both angles?

b) Two angles are supplementary. The measure of one is 24 less than twice the measure of the other. What is the measure of the larger angle?

12. Determine the measure of the complement of an angle whose measure is:

(a) 40

(b) 68

(c) 45.5

(d) n

(e) 45 + n

13. a) Given $m\angle R + m\angle X = 90$ and $m\angle S + m\angle Y = 90$. If $m\angle R > m\angle S$, which of the following is true?

_____ (1) $m\angle X < m\angle Y$

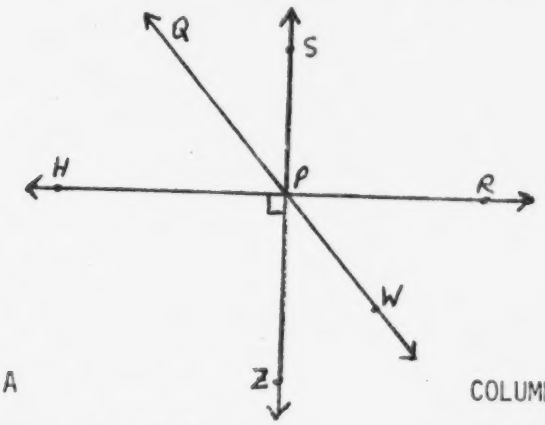
_____ (2) $m\angle X = 90 - m\angle Y$

_____ (3) $m\angle X > m\angle Y$

_____ (4) $m\angle X + m\angle Y = 90$

b) The measure of an angle is 7 times the measure of its complement. Find the measure of the larger angle.

14. Using the illustration below, select the item from Column B that correctly relates to the statement in Column A:



COLUMN A	COLUMN B
____ a) m ∠HPZ + m ∠WPZ = m ∠HPW	1) Angle Measurement Postulate
____ b) m ∠HPZ + m ∠SPR = 180	2) Angle Construction Postulate
____ c) m ∠HPW < 180	3) Angle Addition Postulate
____ d) ∠RPQ and ∠QPH form a linear pair	4) Definition of a Linear Pair
____ e) The measure of ∠SPQ is m ∠SPQ	5) Definition of the Measure of an Angle
	6) Definition of Supplementary Angles
	7) Supplement Postulate

15. a) If you were given that ∠a is the complement of ∠y, ∠b is the complement of ∠x and ∠x ≅ ∠y, what postulate or theorem would you use to prove ∠a ≅ ∠b?

b) Answer True or False:

_____ 1) If $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ form a right angle, then they are perpendicular and intersect at P.

_____ 2) If two lines intersect, then any two adjacent angles form a linear pair.

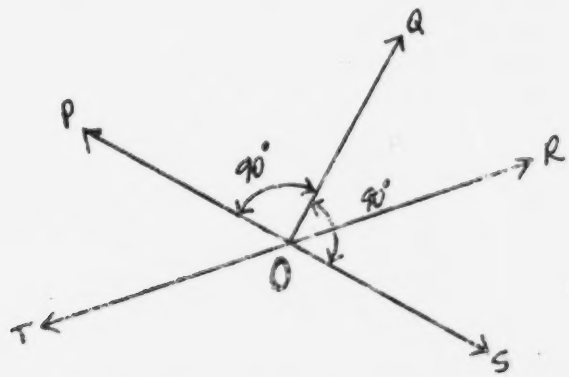
_____ 3) The complement of an acute angle is obtuse.

_____ 4) If two intersecting lines form one right angle, then they form four right angles.

_____ 5) Vertical angles are always right angles or acute angles.

_____ 6) Two angles are vertical angles if their sides form two pairs of opposite rays

16. Use the given figure to name the required angle or angles:



_____ _____ a) Name a pair of equal acute angles.

_____ b) Name a pair of vertical angles which are obtuse.

_____ c) Name a pair of equal angles which form a linear pair.

_____ d) Name a pair of adjacent complementary angles.

_____ e) Name an angle which is a supplement to ∠TOS.

IF YOU HAVE MASTERED THE OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCE STUDY

Section I

APPLICATIONS

Angle of Incidence - the angle between a light ray (incident ray) and the perpendicular (normal) to the point where it meets the reflecting surface (point of incidence).

Angle of Reflection - the angle between the normal and the reflected light ray.

Law of Reflection - a ray of light striking a reflective surface will be reflected so that the angle of reflection is equal to the angle of incidence. In optics, all angles are measured from the normal.

Objective (6) (1) In the Figure I below, will the light ray be reflected to point P, S, or T ?

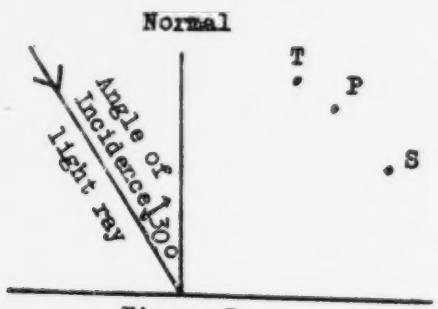


Figure I

Objective (6) (2) Which of the points in Figure II will not receive light from either ray A or ray B ?

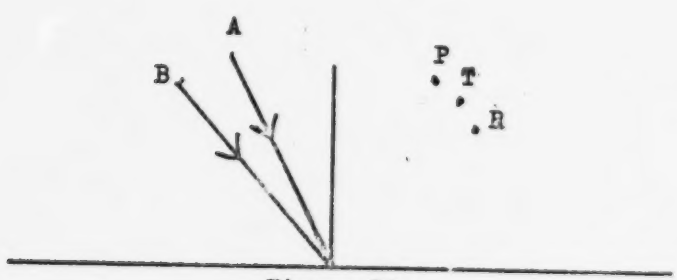
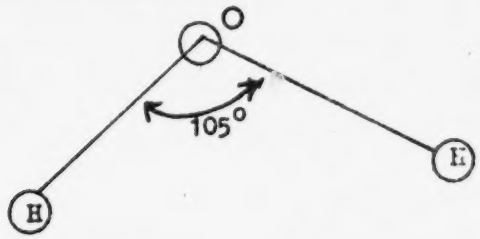


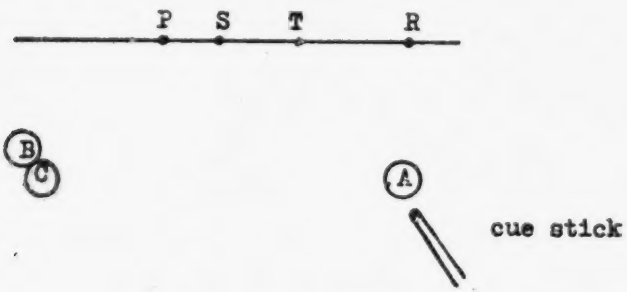
Figure II

Objective (9) (3) In the water molecule, the two hydrogen atoms are on each side of the oxygen atom and form an angle of 105° with the oxygen atom as shown below.



The supplement of this angle indicates the extent to which the molecule departs from being linear. What is the angle for water ?

Objective (14) (4) Three billiard balls are located as shown in the diagram below. Which point will ball A have to hit both ball B and ball C ?



(Cont' on following page)

5. a) Given $\triangle ABC$. A-D-B, B-E-C, C-D-F, and D-G-E.

1) Is G in the interior or exterior of $\triangle ABC$?

2) Does $\overrightarrow{BC}$ intersect $\overrightarrow{AC}$?

3) G and F are on opposite sides of _____?

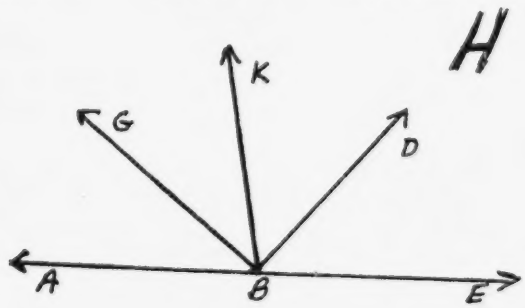
b) Twice the measure of an angle is 30 less than five times the measure of the supplement. What is the measure of the angle?

c) If the sum of the supplement of an angle and three times the complement equals 250^0, how large is the angle?

d) If two straight lines intersect so that one of the smaller angles formed equals 30^0, how large is each of the other angles?

6. a) If, in a plane, m $\angle BAD$ = 65 and m $\angle DAC$ = 32, what is m $\angle CAB$?

b) In half plane H, $\overrightarrow{BA}$ and $\overrightarrow{BE}$ are opposite rays, $\angle ABG \cong \angle KBG$, and $\angle KBD \cong \angle DBE$. Find m $\angle GBD$.



c) If OA, OB, and OC are three distinct rays in a plane such that no two of them are opposite, is each of the following statements true or false? (Recall that just one exception will make the statement false).

_____ (1) m ∠AOB + m ∠BOC = m ∠AOC

_____ (2) m ∠AOB + m ∠BOC = m ∠AOC = 360

If you write that one of the statements is false, justify your answer.

SECTION II

Behavioral Objectives

By the completion of the prescribed course of study, you will be able to:

16. Given a statement, rewrite the statement in the hypothesis-conclusion form. (in a conditional sentence)

17. Given a statement, complete a proof of the statement in two column form using any of the following:

    a) vertical angles are congruent

    b) definition of between

    c) Angle Addition Postulate

    d) supplement of congruent angles are congruent

    e) complement of congruent angles are congruent

Resources

I. Readings:

1. Moise: Geometry - #16 - #17 pp. 95-98.
2. Jurgensen: Modern Geometry - #16 - #17 p. 91.
3. Nichols: Modern Geometry - #16 - #17 pp. 76-77.
4. Anderson: School Mathematics Geometry - #16 - #17 pp. 82-83.

II. Problems:

1. Moise: Geometry - #16 - #17 p. 96 ex. 1-2, pp. 98-99 ex. 1-8.
2. Jurgensen: Modern Geometry - #16 - #17 pp. 92-93 ex. 1-16, p. 130 ex. 21-28, p. 136 ex. 23-32, pp. 142-143 ex. 21-37.
3. Nichols: Modern Geometry - #16 - #17 p. 78 ex. 1-3.
4. Anderson: School Mathematics Geometry - #16 - #17 pp. 83-84 ex. 1-18, p. 124 ex. 8-9, 12, p. 129 ex. 7-10, 12.

SELF-EVALUATION II

1. Rewrite each of the following below in "If _____ then _____" form:

   a) Supplements of congruent angles are congruent.

   b) The intersection of two planes is a line.

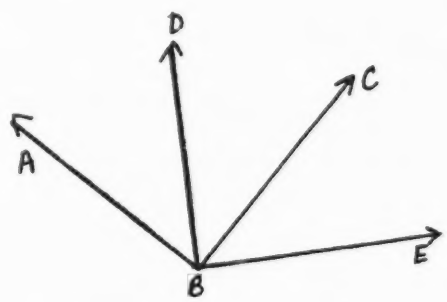
   c) Vertical angles are congruent.

   d) Congruent angles have the same measure.

   e) Two angles with the same measure are congruent.

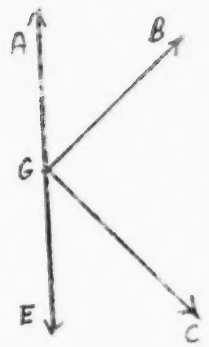
2. Complete the proof by filling in the blanks:

Given: The figure with
m ∠ABC = 90 = m ∠DBE
Prove: ∠ABD ≅ ∠CBE



PROOF: Statements	REASONS
1. m ∠ABC = 90 = m ∠DBE	1. Given
2. ∠ABD is complementary to ∠DBC	2. _____
3. _____	3. Definition of Complementary Angles
4. ∠DBC ≅ ∠DBC	4. _____
5. _____	5. _____

3. Complete the proof by filling in the blanks:



Given: The figure with $\overrightarrow{GA}$ opposite to $\overrightarrow{GE}$ and $\overrightarrow{GB} \perp \overrightarrow{GC}$.

Prove: $\angle AGB$ is complementary to $\angle EGC$.

PROOF: Statements	REASONS
1. $\overrightarrow{GA}$ is opposite to $\overrightarrow{GE}$	1.
2. $\angle AGB$ is supplementary to $\angle BGE$	2.
3. $m\angle AGB + m\angle BGE = 180$	3.
4. $\overrightarrow{GB} \perp \overrightarrow{GC}$	4.
5. $m\angle BGC = 90$	5.
6. $m\angle BGE = m\angle EGC + 90$	6.
7. $m\angle AGB + m\angle EGC + 90 = 180$	7.
8. $m\angle AGB + m\angle EGC = 90$	8.
9. $\angle AGB$ is complementary to $\angle EGC$	9.

4. Supply a complete proof of the following:

Given: WY = XZ
Prove: WX = YZ

SELF-EVALUATION II (cont')

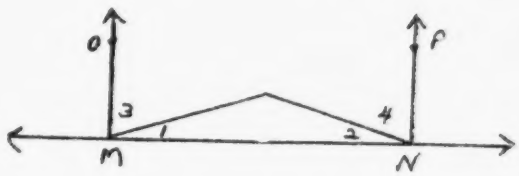
5. Supply a complete proof of the following:

Given: $\overrightarrow{MO}$ $\overleftrightarrow{MN}$

$\overrightarrow{NP} \perp \overleftrightarrow{MN}$

$\angle 1 = \angle 2$

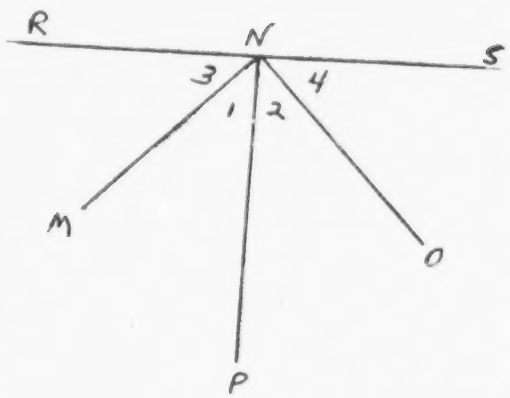
PROVE: $\angle 3 = \angle 4$



IF YOU HAVE MASTERED ALL THE BEHAVIORAL OBJECTIVES, A PROGRESS TEST IS SCHEDULED. AFTER THE PROGRESS TEST, A LAP TEST IS SCHEDULED.

ADVANCE STUDY II

1. Write a complete proof for the following:

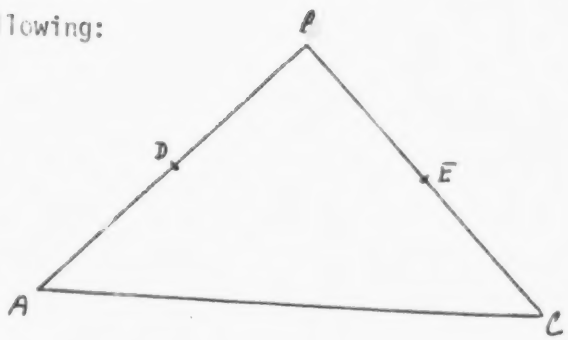


Given: $\overrightarrow{NP}$ bisects $\angle MNO$

$\overleftrightarrow{RNS}$ $\quad$ $\overrightarrow{NP}$

Prove: $\angle 3 \cong \angle 4$

2. Write a complete proof for the following:

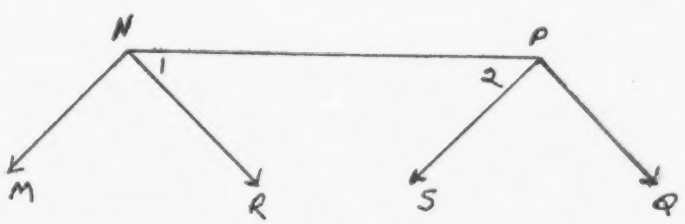


Given: In $\triangle ABC$, $\overline{AB} = \overline{BC}$;

D bisects $\overline{AB}$; E bisects $\overline{BC}$.

Prove: $\overline{AD} \cong \overline{CE}$

3. Write a complete proof for the following:



Given: $\angle MNP \cong \angle NPQ$

$\overrightarrow{NR}$ bisects $\angle MNP$

$\overrightarrow{PS}$ bisects $\angle NPQ$

Prove: $\angle 1 \cong \angle 2$

BIBLIOGRAPHY

I. Textbooks:

1. Moise, Downs - Geometry (Addison-Wesley Publishing Co., Inc., 1967).

2. Jurgensen, Donnelly, Dolciani - Modern Geometry (Houghton Mifflin Co., 1965).

3. Nichols, Palmer, Schacht, Spencer - Modern Geometry (Holt Rinehart Winston, Inc., 1968).

4. Anderson, Garon, Germillion - School Mathematics Geometry (Houghton Mifflin Co., 1969).

5. Lewis - Geometry, A Contemporary Course (D. Van Nostrand Co., Inc., 1968).

LAP

LEARNING
ACTIVITY
PACKAGE

CONGRUENCES

Ninety Six High School
Ninety Six, S.C.

Geometry 114

LAP NUMBER 29

WRITTEN BY Bill Holland

EWED BY

RATIONALE

In this section we explore the concepts of congruence, particularly the congruence relation in the set of triangles. Classical theorems about congruent triangles, angle bisectors, overlapping triangles and isosceles triangles will be introduced.

It is most important that you should develop some ability in proving theorems. Elementary congruence theorems offer an ideal setting for moving from geometric intuition to a certain rigor. You will be introduced to a variety of important aspects of proof and should become wary of "circular reasoning".

The purpose of a proof is essentially to convince someone of the validity of the mathematical proposition under consideration. Our primary concern will be with the organization of mathematical proofs by a deductive system.

You will find that the skills gained in analyzing and proving mathematical propositions will re-enforce your ability to objectively evaluate procedures necessary to a scientific experiment.

SECTION I

Behavioral Objectives

By the completion of the prescribed course of study, you should be able to:

1.. Given any pair of geometric figures, write a correspondence between the vertices which is a congruence.

2. Given any pair of congruent triangles, write a correspondence which is a congruence:

   a. between triangles
   b. between pairs of segments
   c. between pairs of angles.

3. Answer correctly true-false, multiple choice or matching type questions relating to:

   a. identity congruences
   b. congruent angles and equal angles
   c. congruent segments and equal segments
   d. included sides of a triangle
   e. included angles of a triangle
   f. isosceles triangles
   g. equilateral triangles
   h. angle bisectors
   i. bisectors of an angle of a triangle
   j. medians of a triangle
   k. altitudes of a triangle
   l. squares and rectangles

4. Given a drawing of one or more triangles, and the necessary information, name:

   a. the angles included by two sides
   b. the side included by two angles

5. Given any pair of congruent triangles, name the congruence between these triangles by applying the correct postulate:

   a. S.A.S. Postulate
   b. A.S.A. Postulate
   c. S.S.S. Postulate

6. Given a suitable statement, or appropriately marked figure, write the hypothesis and conclusion as "given" and "to prove".

SECTION I

Resources

I. Readings:

1. Moise: #1 pp. 105-107; #2-#4 pp. 112-115, 143-145; #5-#6 pp. 119-121, 128-129.
2. Jurgensen: #1 p. 191; #2- #4 pp. 41, 59, 65, 190-191, 213-214; #5-#6 pp. 193-195, 198-199, 91-92.
3. Anderson: #1 pp. 145-147; #2-#4 pp. 151-153, 178-179, 310-311; #5-#6 pp. 154-160, 168-171.
4. Lewis: #1 pp. 110-116; #2-#4 pp. 118-123; #5-#6 pp. 123, 153-155, 164-165.
5. Nichols: #1 pp. 144-146; #2-#4 pp. 148-149, 163-164, 168, 236-238, 257, 295; #5-#6 pp. 150-151, 154-155.

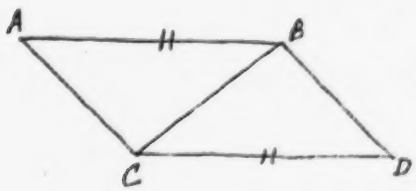
II. Problems:

1. Moise: #1 pp. 108-110 ex. 1-9; #2-#4 pp. 115-117 ex. 1-12, p. 145 ex. 1; #5-#6 pp. 121-122 ex. 1-2, p. 129 ex. 1-2.
2. Jurgensen: #1-#4 pp. 59-60 ex. 1, 9, p. 65 ex. 1-4, pp. 192-193 ex. 1-14, p. 195 ex. 1, 2, 4; #5-#6 p. 195 ex. 5, 6, 9-11, 13-14, 16, p. 92 ex. 1-10.
3. Anderson: #1 pp. 148-151 ex. 1-10; #2-#4 pp. 154-155 ex. 1-4, 7-13, p. 157 ex. 2, p. 179 ex. 1-4; #5-#6 p. 161 ex. 1.
4. Lewis: #1 pp. 116-117 ex. 1-8; #2-#4 ____; #5-#6 pp. 123-125 ex. 1-10, pp. 125-126 ex. 1-12.
5. Nichols: #1 pp. 146-147 ex. 1-5; #2-#4 ____; #5-#6 p. 152 ex. 1, p. 156 ex. 1.

SELF-EVALUATION I

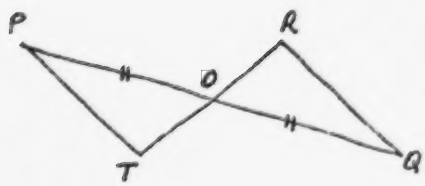
1. For each of the figures, complete the correspondence between the vertices which is a congruence for the pairs of congruent figures as indicated:

A.



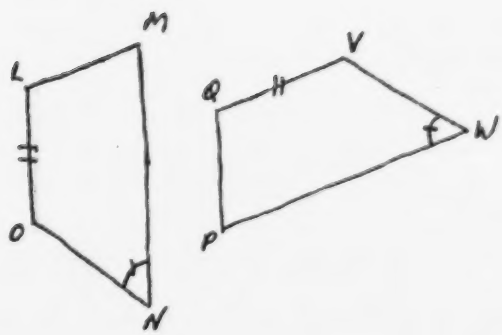
CAB ↔ ?

B.



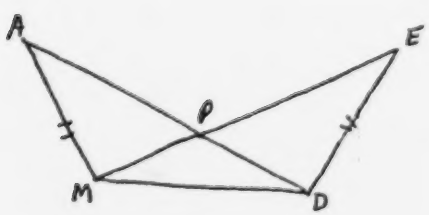
QOR ↔ ?

C.



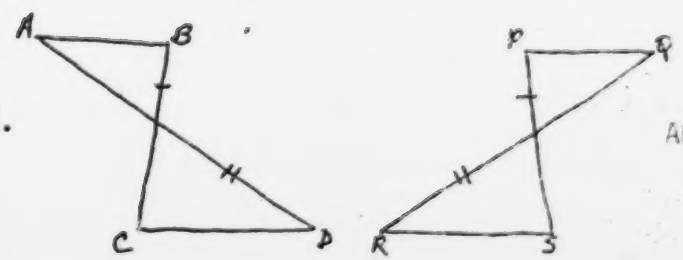
NOLM ↔ ?

D.



MAD ↔ ?

E.

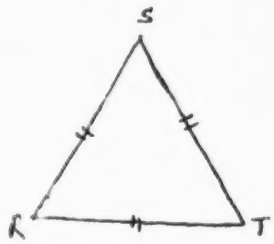


ABCD ↔ ?

2. For each figure given below, list all the correspondences between the vertices for pairs of congruent triangles:

a)

b)



3. Below are listed six pairs of corresponding parts of two congruent triangles. Name the congruent triangles:

$\overline{AB} \cong \overline{MK}$ $\quad\quad$ $\angle A \cong \angle M$

$\overline{BW} \cong \overline{KF}$ $\quad\quad$ $\angle B \cong \angle K$

$\overline{AW} \cong \overline{MF}$ $\quad\quad$ $\angle W \cong \angle F$

4. Given $\triangle PQN \cong \triangle RXW$. Name the six pairs of corresponding parts.

5. Answer the following as true or false. If false, tell why.

_____ a) In $\triangle JFK$, $\angle K$ is included between $\overline{JK}$ and $\overline{JF}$.

_____ b) Every angle has exactly one bisector.

_____ c) Two segments are equal if they have the same length.

_____ d) If D is in the interior of $\angle TAB$, then $\overrightarrow{AD}$ bisects $\angle TAB$.

_____ e) Every equilateral triangle is congruent.

_____ f) A triangle is congruent to itself.

_____ g) In $\triangle PST$, $\overline{PS}$ is included by $\angle P$ and $\angle T$.

_____ h) The median of a triangle bisects and angle of the triangle.

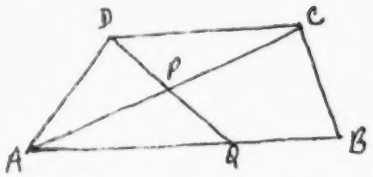
_____ i) All isosceles triangles are equilateral.

_____ j) If $\triangle ADM \cong \triangle BEN$, then $\overline{MD} = \overline{EN}$ and $\angle D = \angle N$.

_____ k) All rectangles are squares.

_____ l) If $\overline{AD}$ is a median of $\triangle ABC$, then $BD = DC$.

6. Given the figure as shown. List the part which correctly fills the blank.



a) side, angle, side of $\triangle ACD$: $\overline{AC}$, ____, $\overline{AD}$

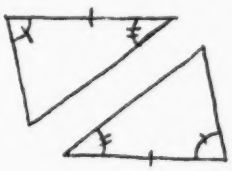
b) angle, side, angle of $\triangle ABC$: ____, $\overline{AB}$, ____

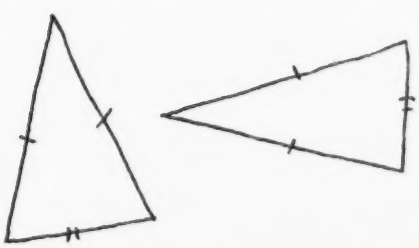
c) side, angle, side of $\triangle PAD$: ____, $\angle DPA$, ____

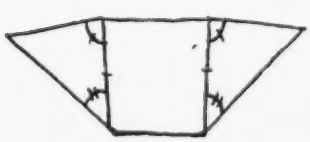
d) angle, side, angle of $\triangle CDP$: $\angle CDP$, ____, $\angle CPD$

e) angle, side, angle of $\triangle DAQ$: $\angle DAQ$, ____, $\angle DQA$

7. If for each of the pairs of triangles sketched below, like markings indicate congruent parts, name the congruence postulate which will prove the triangles congruent. If none, write none.

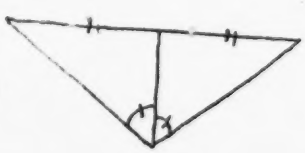
a) 

b) 

c) 

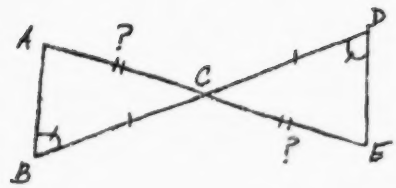
d)

e)

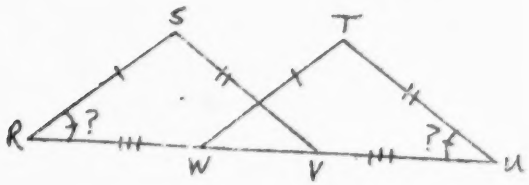
f) 

8. Each pair of fiugres given below are drawn so that the hypothesis and conclusion are specified. Write the "give" and "to prove" for each pair:

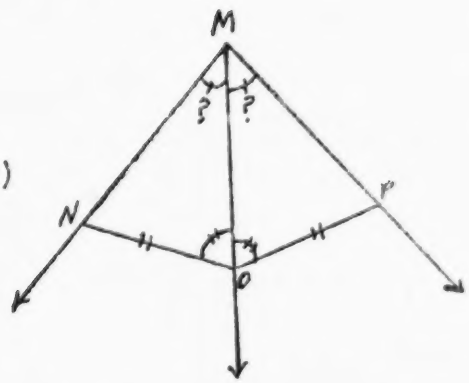
a)



b)



c)



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST.

ADVANCE STUDY I

1. Construct the triangles determined by each triple of measures listed below. If two triangles may be constructed from each of the measures construct both. If more than two triangles or no triangles may be constructed, explain why.

   a) $m\angle A = 60$, $AB = 4$, $m\angle B = 90$
   b) $m\angle X = 110$, $AB = 10$, $BC = 6$
   c) $m\angle W = 18$, $WU = 3$, $TU = 2$
   d) $MN = 10$, $NO = 6$, $AO = 8$
   e) $m\angle R = 60$, $RS = 4$, $m\angle S = 120$
   f) $MN = 3$, $NO = 4$, $MO = 8$
   g) $AB = 4$, $BC = 8$, $m\angle A = 60$
   h) $m\angle M = 30$, $m\angle N = 130$, $m\angle O = 20$

2. For ΔXYZ, $\Delta XYZ \cong \Delta YXZ$ and $\Delta XYZ \cong \Delta ZYX$, what can you deduce about ΔXYZ? Justify your answer.

3. Given $AB \perp XY$ with $X - P - Y$. Points R AND S are on the side of $\overleftrightarrow{XY}$ as B, but R and S are on opposite sides of $\overleftrightarrow{AB}$. R is on the same side of $\overleftrightarrow{AB}$ as X. $\Delta RBA \cong \Delta SBA$. Prove that $\angle XAR \cong \angle YAS$.

4. An equivalence relation is defined as a relation among the members of a set which has the following properties: If a, b, and c are any members of the set, then

   1) $a * a$ (Reflexive)
   2) If $a * b$, then $b * a$ (Symmetric)
   3) If $a * b$ and $b * c$, then $a * c$ (Transitive)

   In applying this relation, you should replace the asterisk by the relation.

   Choose an appropriate set for the following relations and then determine which are equivalence relations.

   a) is less than
   b) is equal to
   c) is the reciprocal of
   d) is a classmate of
   e) is taller than
   f) is greater than

   Is congruence an equivalence relation? Write a supporting argument to justify your answer.

SECTION II

BEHAVIORAL OBJECTIVES

By the completion of the prescribed course of study, you will be able to:

7. Given any triangle, apply the appropriate definitions, theorems, and corollaries, to prove statements relating to isosceles or equilateral triangles.
8. Given any geometric figure, a hypothesis and conclusion, with an incomplete two-column proof concerning a congruence, complete the proof by using the appropriate definitions, postulates and theorems:
   a) Identity Congruence Theorems
   b) Definition of an angle bisector
   c) Definition of a bisector of any given angle of a triangle
   d) Definition of a median
   e) Definition of an isosceles triangle
   f) Definition of an equilateral triangle
   g) Theorems relating to isosceles and equilateral triangles
   h) Definition of a square
   i) Definition of a rectangle
   j) Congruence of triangles postulates
9. Given a hypothesis and a required conclusion involving a congruence between overlapping triangles and non-overlapping triangles, use the appropriate definitions, theorems, and postulates to write an original proof for this conclusion.

SECTION II

RESOURCES

I. Readings:

1. Moise: #7 pp. 134-136; #8 - #9 pp. 122-125, 132, 134-136, 138-140, 143-144.
2. Jurgensen: #7 pp. 207-210; #8 - #9 pp. 201-202, 204.
3. Anderson: #7 pp. 173-175; #8 - #9 pp. 164-166, 177-179.
4. Lewis: #7 pp. 141-149; #8-#9 pp. 126-128, 131-132, 136, 139.
5. Nichols: #7 pp. 153-154; #8 - #9 pp. 151, 158.

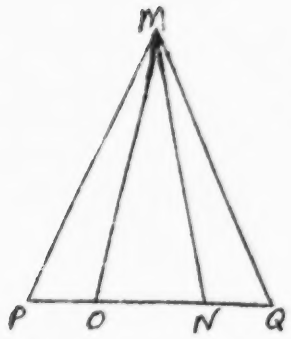
II. Problems:

1. Moise: #7 pp. 136-137 ex. 1-15; #8 - #9 pp. 125-127 ex. 1-10, pp. 130-131 ex. 3-14, p. 133 ex. 1-5, 7; pp. 136-137 ex. 1-12, 14, 15, pp. 141-143 ex. 1-6, 9-10, 13, 15, 18-19, pp. 145-146 ex. 1-11, pp. 147-148 ex. 1-12.
2. Jurgensen: #7 pp. 210-212 ex. 1-26; #8 - #9 pp. 203 ex. 1-17, pp. 205-207 ex. 1-32.
3. Anderson: #7 pp. 176-177 ex. 1-16; #8 - #9 pp. 162-164 ex. 2-12, pp. 167-178 ex. 1-10, pp. 171-172 ex. 1-14, pp. 180-181 ex. 1-11.
4. Lewis: #7 pp. 149-152 ex. 1-22; #8 - #9 pp. 129-131 ex. 1-16, pp. 133-135 ex. 1-16, pp. 137-138 ex. 1-10, pp. 140-141 ex. 1-12, pp. 158-159 ex. 1-12, pp. 166-167 ex. 1-8, 10, 12.
5. Nichols: #7 p. 169 ex. 1-2; #8 - #9 p. 152 ex. 2-8, p. 157 ex. 2-16, p. 159 ex. 1-3, pp. 165-166 ex. 1-7, p. 169 ex. 3-7.

SELF-EVALUATION II

1. Given $\overline{MN} \cong \overline{MO}$
   $\overline{PN} \cong \overline{QO}$

   Prove: $\angle PMO \cong \angle QMN$

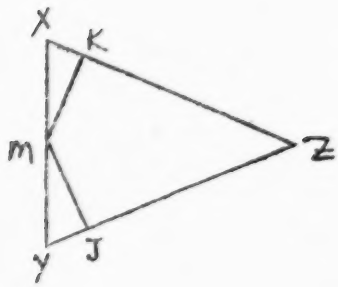


2. Given $\triangle XYZ$; $\overline{MK} \perp \overline{XZ}$

   $\overline{MJ} \perp \overline{YZ}$, $\overline{XK} \cong \overline{YJ}$

   $\overline{MK} \cong \overline{MJ}$

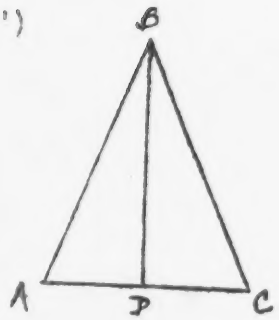
   Prove: $\triangle XYZ$ is isosceles.



3. For the following statement:

   a) Write it in the hypothesis-conclusion form.
   b) Identify the "given" and "to prove".
   c) Draw a figure and write out a proof of the statement:

   "The bisector of the vertex angle of an isosceles triangle is perpendicular to the base."



4. Complete the following proof:

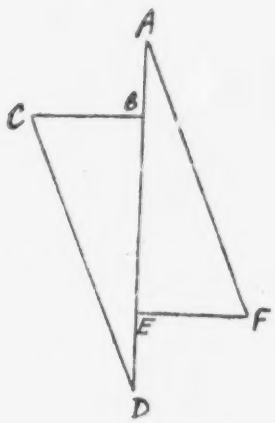
Given: D is the midpoint of $\overline{AC}$,
$\overline{BA} \cong \overline{BC}$ and $\angle A \cong \angle C$

Prove: $\angle ABD \cong \angle CBD$

PROOF: STATEMENT	REASON
1. D is the midpoint of $\overline{AC}$	1.
2. AD = DC	2.
3. BA = BC	3.
4. m $\angle A$ = m $\angle C$	4.
5.	5.
6.	6.

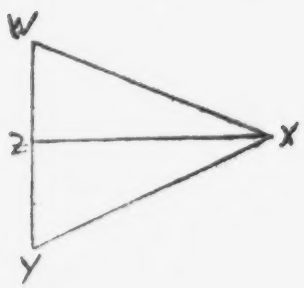
5. Given: AB = DE, FE = CB
$\overline{FE} \perp \overline{AD}$, $\overline{CB} \perp \overline{AD}$

Prove: $\angle F \cong \angle C$



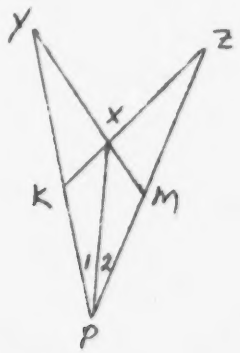
6. Given: $\overline{WX} \cong \overline{YX}$
Z is the midpoint of $\overline{WY}$

Prove: $\overrightarrow{XZ}$ is the bisector of $\angle WXY$.



7. Given: ∠1 = ∠2, KP = MP

   Prove: YX = ZX



IF YOU HAVE MASTERED THE BEHAVIORAL OBJECTIVES, TAKE YOUR PROGRESS TEST. A LAP TEST IS SCHEDULED AFTER THE PROGRESS TEST.

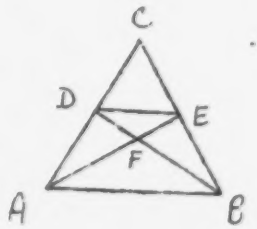
ADVANCE STUDY II

1. a) Given that $\overline{BQ}$ bisects $\overline{PA}$ at R, but BQ $\neq$ PA. B and Q are on opposite sides of $\overleftrightarrow{PA}$. S and C are points on $\overline{PR}$ and $\overline{AR}$, respectively, such that RS = RC, $\overline{BC} \perp \overline{PA}$ and $\overline{QS} \perp \overline{PA}$. Also, $\angle BAR \cong \angle QPR$. Prove that $\overline{PA}$ bisects $\overline{BQ}$ and that $\angle ABC \cong \angle PQS$.

   b) Given $\angle HRE$ with RH = RE. Points M and K are on the sides of $\angle HXE$ such that R - H - M and R - E - K. $\overline{EM}$ and $\overline{HK}$ intersect at T. $\angle HRT \cong \angle ERT$. Prove that $\triangle MTH \cong \triangle KTE$.

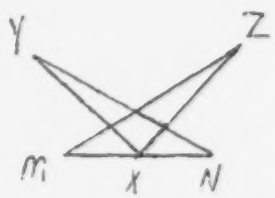
2. a) Given: AC = BC, DC = EC

   Prove: DF = EF



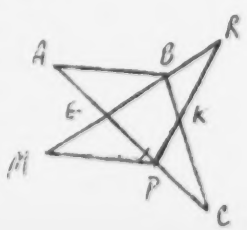
   b) Line L is perpendicular to $\overline{XY}$ and bisects $\overline{XY}$ at S. Points R and T are the midpoints of $\overline{XS}$ and $\overline{YS}$, respectively. Points A and B are taken on L on opposite sides of $\overleftrightarrow{XY}$ such that AX = BY and AT = BR. Prove that AS = BS.

   c) In the figure, if M, N, X, Y, and Z are coplanar, X is the midpoint of $\overline{MN}$, $\angle M \cong \angle N$ and $\angle MXY \cong \angle NXZ$, prove that $\angle Y \cong \angle Z$.



   d) Must M, N, X, Y, and Z be coplanar in (c)? Justify your answer.

3. a) In the figure given, points G and B trisect $\overline{MR}$ and points G and P trisect $\overline{AC}$. If AG = BG show that $\angle R \cong \angle C$.



3. b) If $\overline{XY}$ is perpendicular to each of three different rays $\overrightarrow{XA}$, $\overrightarrow{XB}$, and $\overrightarrow{XC}$, and XA = XB = XC, prove that AY = BY = CY.

c) Let L be the edge of two half-planes H_1 and H_2. A and B are two points of L, M is a point in H_1, and R is a point in H such that $\angle MAB \approx \angle RAB$ and AM = AR. Prove $\triangle MRB$ is isosceles.

d) Must $\overline{MR}$ intersect L in (c)?

e) Does tha answer to (c) require that H_1 and H_2 be coplanar?

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